Learning Three-Dimensional Flow for Interactive Aerodynamic Design

https://youtu.be/U38cKk-sxyY

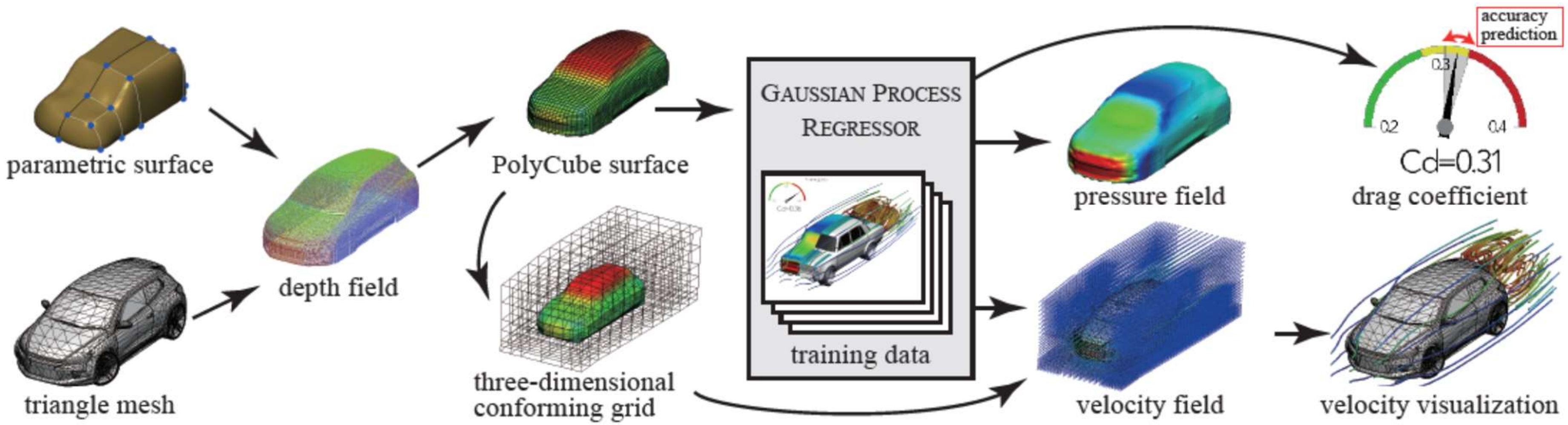
http://pub.ist.ac.at/~bbickel/downloads/2018_sigg_Learning3DAerodynamics.pdf

http://www.jaist.ac.jp/~xie/panel.html

Applications

Learning Three-Dimensional Flow for Interactive Aerodynamic Design

Machine learning framework which predicts aerodynamic forces and velocity and pressure fields given a three dimensional object shape and Reynolds number input.



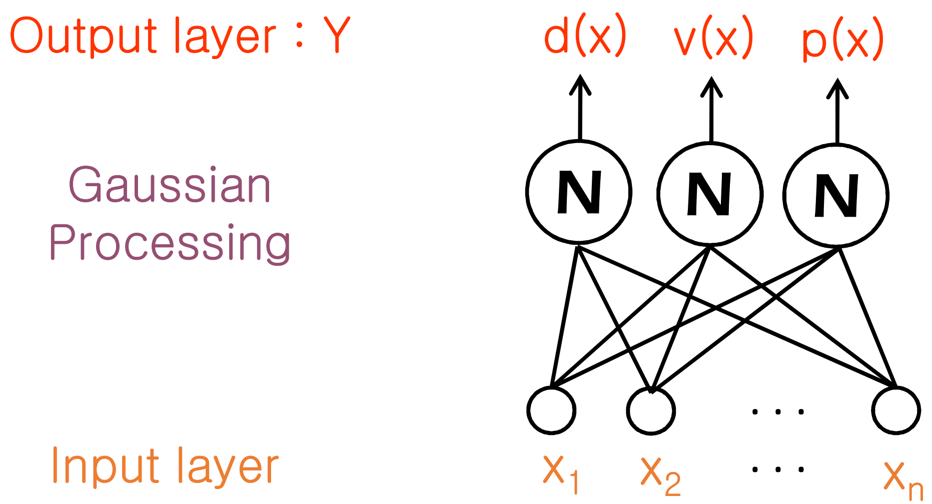
Applications

Learning Three-Dimensional Flow for Interactive Aerodynamic Design

Gaussian Process (GP) regression for inferring the CFD simulation data

Three regressors : for drag coefficient, non-dimensionalized velocity, and pressure.
Input : Parametric modeling vector of car + Reynolds No.

Output data set : $X = \{y_1 y_2 ... y_N\}$



$$\mathbf{y} \sim N(\mu(x;\theta), \sigma^2)$$
$$X = \{x_1, \dots, x_n\}, x_i \in R^n, i = \overline{1, N}$$
$$Y = \{y_1, \dots, y_n\}, y_i \in R, i = \overline{1, N}$$

Input data set : $X = \{\vec{x}_1 \vec{x}_2 \ldots \vec{x}_N\}$

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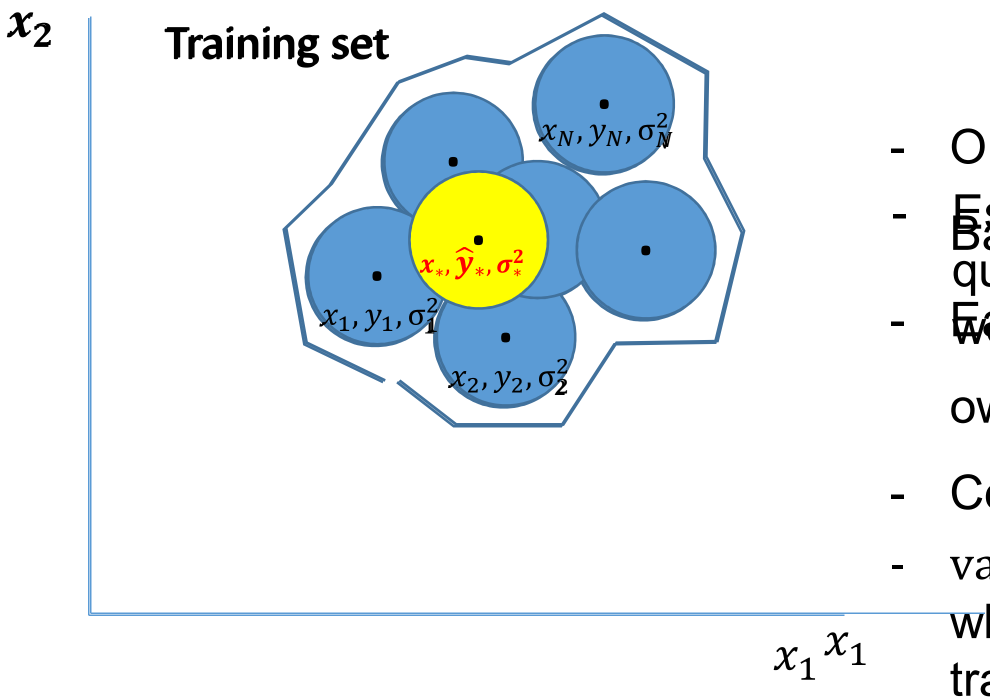
Gaussian Process (GP) regressor ~ **Radial Basis Function** Regressor !



$$\hat{y}_* = \sum_{i=1}^{N} y_i \exp\left(-\frac{\|\vec{x}_* - \vec{x}_i\|^2}{2\sigma_i^2}\right)$$

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- Output can be estimated using Radial Basis Function of training set(D)
- Each vector of training set(D), $\vec{x}_i$, has it's own radial basis
- Center of radial basis is $\vec{x}_i$
- vairiance σ_i^2 would be set to the value which minimizes the MSE on the training set

- Estimated Output($\hat{y}_*$) for a given query($\vec{x}_*$) has Gaussian distribution as well.

➔ **Uncertainty !**

$$\hat{y}_* = \mathbf{N}(\mu_*, \sigma_*^2)$$

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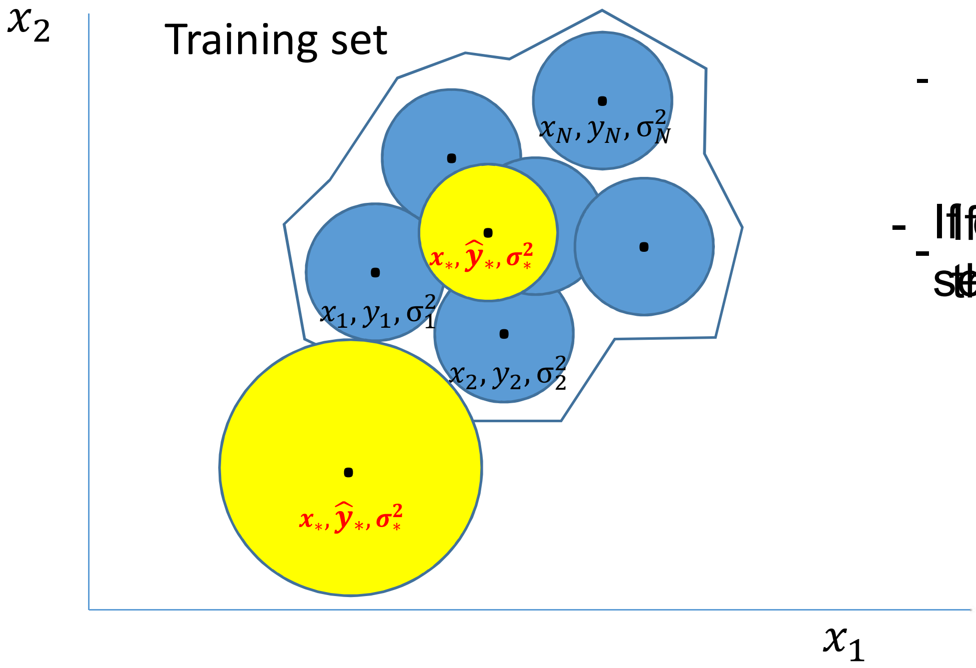

$$\hat{y}_* = \mathbf{N}\,(\mu_*, \sigma_*^2)$$

- Output variance(σ_*^2) is related with Covariance with Training set
- If query is inside of cluster of training set, then σ_*^2 is small one.
- If query is outside of cluster of training set, then σ_*^2 is large one.
- Covariance is a metric of similarity or correlation between two vectors

$$\begin{pmatrix} \mathbf{f} \\ \mathbf{f}_* \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \boldsymbol{\mu} \\ \boldsymbol{\mu}_* \end{pmatrix}, \begin{pmatrix} \mathbf{K} & \mathbf{K}_* \\ \mathbf{K}_*^T & \mathbf{K}_{**} \end{pmatrix} \right)$$

$$\begin{aligned} p(\mathbf{f}_*|\mathbf{X}_*, \mathbf{X}, \mathbf{f}) &= \mathcal{N}(\mathbf{f}_*|\boldsymbol{\mu}_*, \boldsymbol{\Sigma}_*) \\ \boldsymbol{\mu}_* &= \boldsymbol{\mu}(\mathbf{X}_*) + \mathbf{K}_*^T\mathbf{K}^{-1}(\mathbf{f} - \boldsymbol{\mu}(\mathbf{X})) \\ \boldsymbol{\Sigma}_* &= \mathbf{K}_{**} - \mathbf{K}_*^T\mathbf{K}^{-1}\mathbf{K}_* \end{aligned}$$