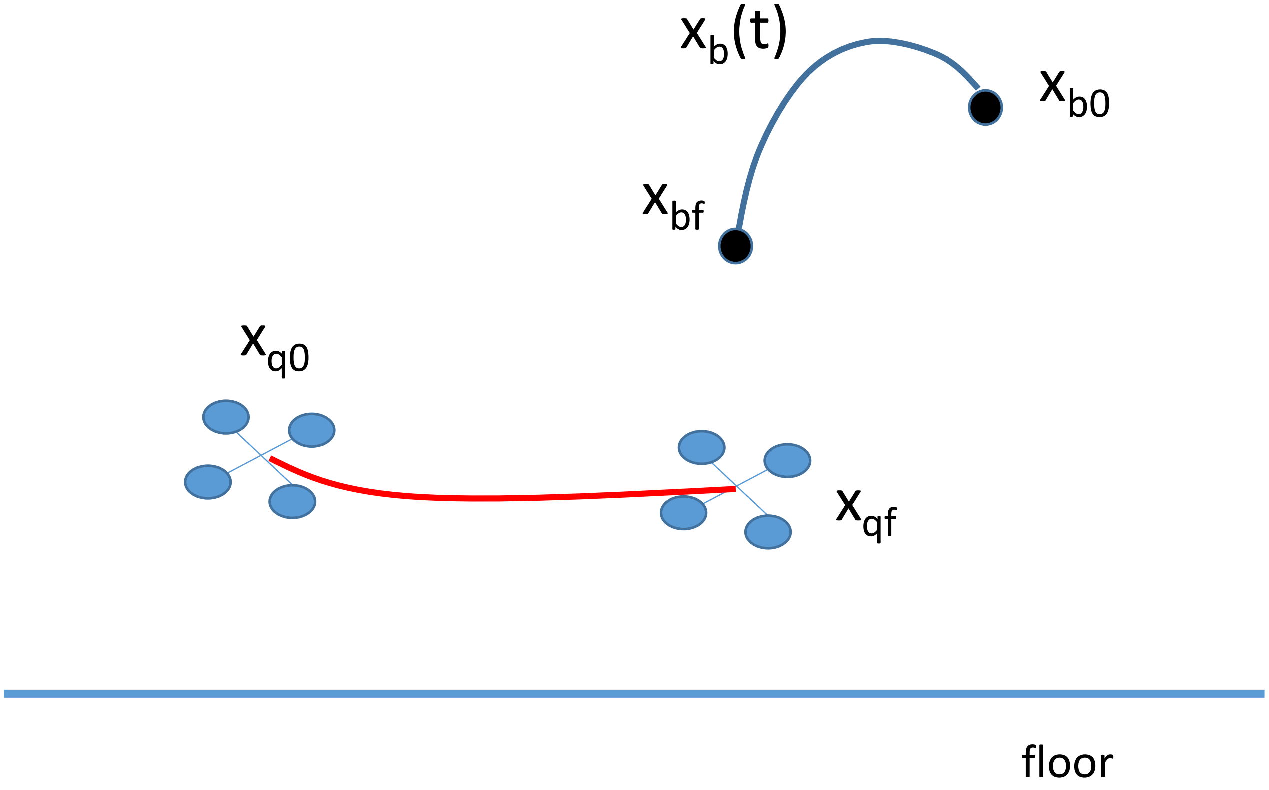
Watch the video clip

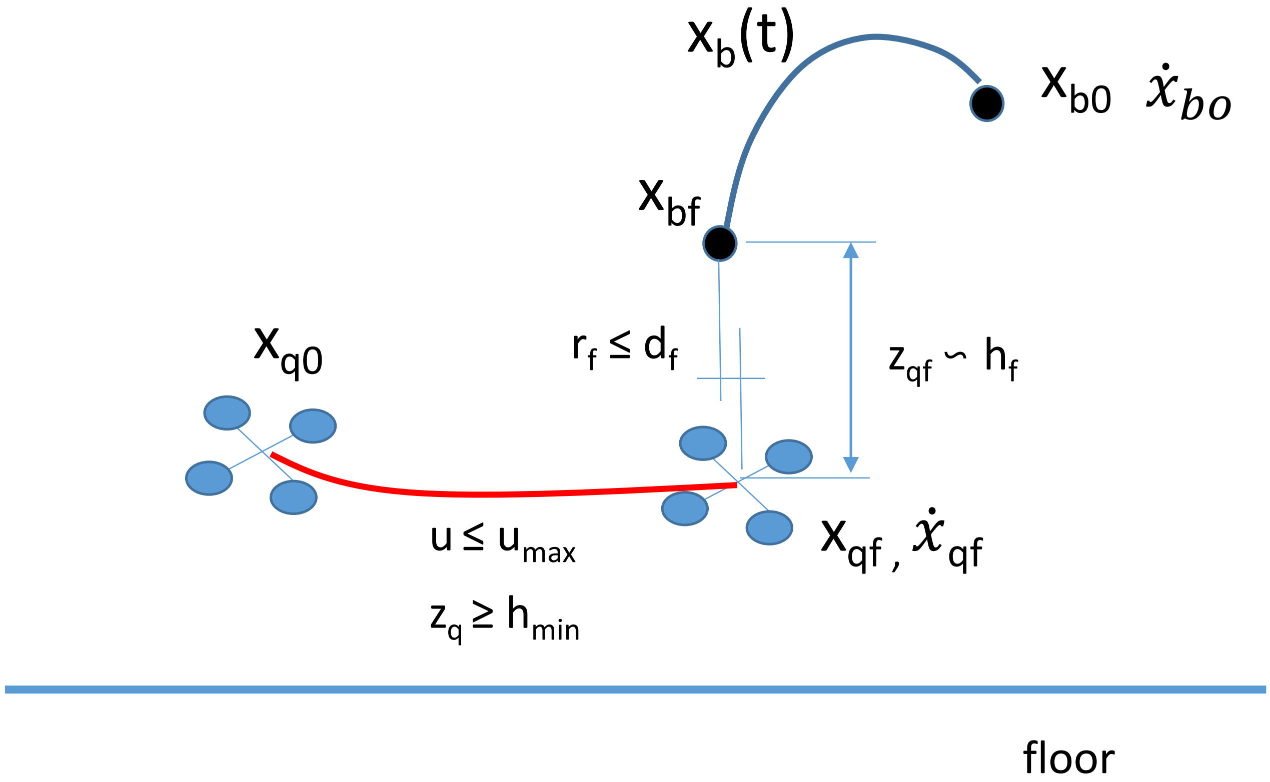
“ A computationally efficient motion primitive for Quadrocopter trajectory generation”

https://youtu.be/oMy5y-eQVeE

Imagine Ball Catching with Quadcopter



Imagine Ball Catching with Quadcopter



Imagine Ball Catching with Quadcopter

Eq. of motion : $\dot{X}_q = f(Xq, u)$

Initial condition : x_{q0} x_{b0} $\dot{x}_{bo}$

Terminal condition : $r_f \leq d_f$ $z_{qf} \backsim h_f$

Constraint :

$u \leq u_{max}$

$z_q \geq h_{min}$

Optimal Control !

How to design cost function ?

Minimum energy consumption or minimum time with constraints ?

Objectives of the optimal control :

- Minimization of the error, $E(x(t_f))$: $(r_f - d_f)^2, (z_{qf} - h_f)^2, (\dot{x}_{qf})^2$ **Cost @ terminal**
- Minimization of energy, $\int_{t_0}^{t_f} L\big(x(t), u(t)\big)dt$: $\int_{t_0}^{t_f} |u(t)|dt$
- Minimization of energy & time, $\int_{t_0}^{t_f} L\big(x(t), u(t)\big)dt$: $\int_{t_0}^{t_f} (1 + b|u(t)|)dt$

Integration of cost rate during flight

minimize $x(\cdot), u(\cdot)$ $\quad \int_0^T L(x(t), u(t))\, dt \; + \; E(x(T))$

subject to

Eq. of motion : $\dot{X}_q = f(Xq, u)$

Initial condition : x_{q0} x_{b0} $\dot{x}_{bo}$

Constraint : $u \leq u_{max}$

$z_q \geq h_{min}$

Data-Driven Control with Machine Learning

Prof. Steve Brunton, Univ. of Washington

Challenges :

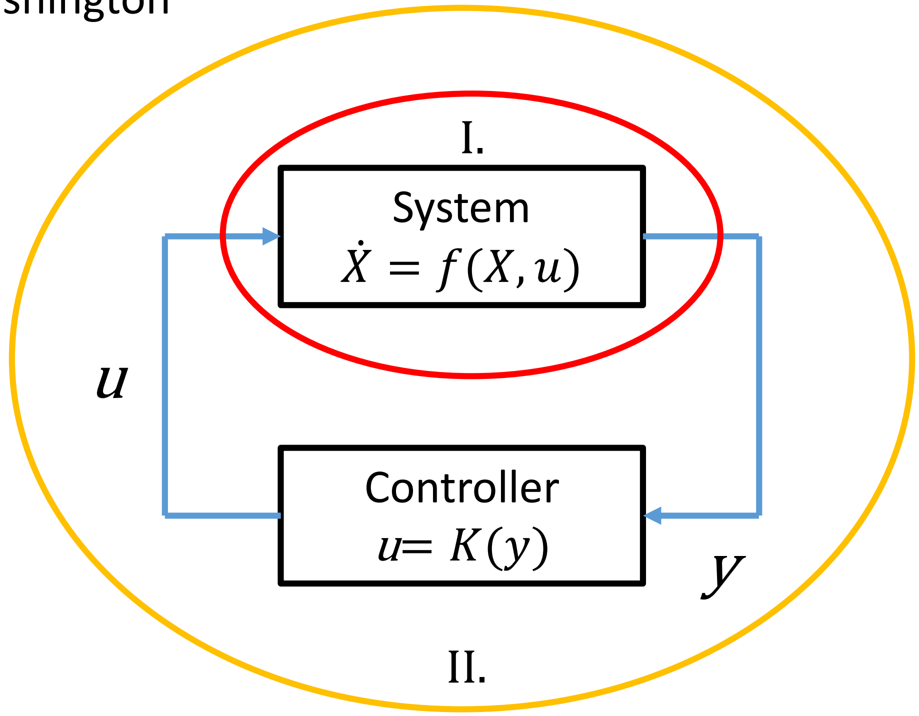
- Nonlinear
- Unknown Dynamics
- High Dimensional

What is Control ? :

- Optimization constrained by dynamics

What is Machine Learning ?

- Powerful NL optimization tool based on data



I. Data Driven Models : System Id.

II. Determine Optimal Control Polity

Improving a Quadrotor Model using Flight Data

Modeling Dynamic Systems for

Multi-Step Prediction with

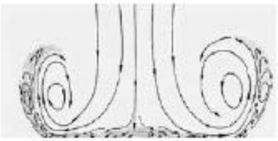
Recurrent Neural Networks

Nima Mohajerin, Uni. Of Waterloo

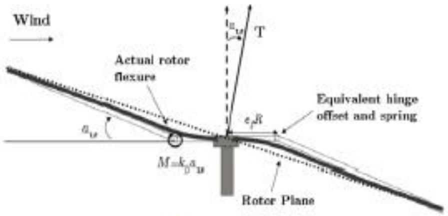
Motivation

Predicting the behaviour of a dynamic system has always been a challenging and important problem in engineering.

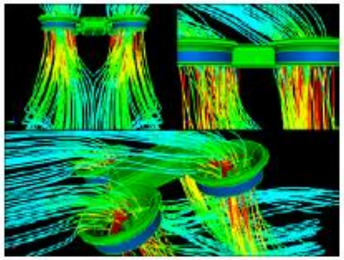
- Complex nonlinearities, e.g. :



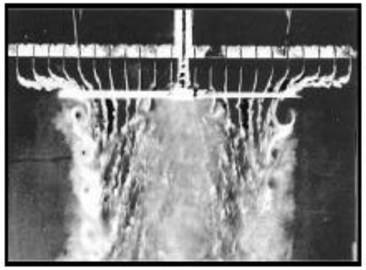
ground effect



blade flapping



aerodynamic effects

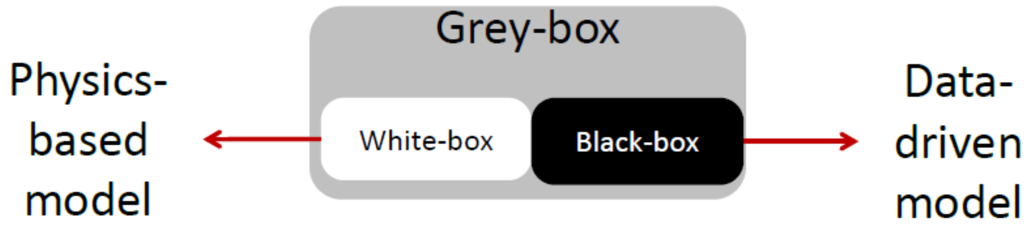


vortex effect

- Simple changes to a quadrotor physical characteristics (payload modification, changing actuators, etc.) may cause significant change in the dynamical model that requires additional modeling efforts.

Motivation

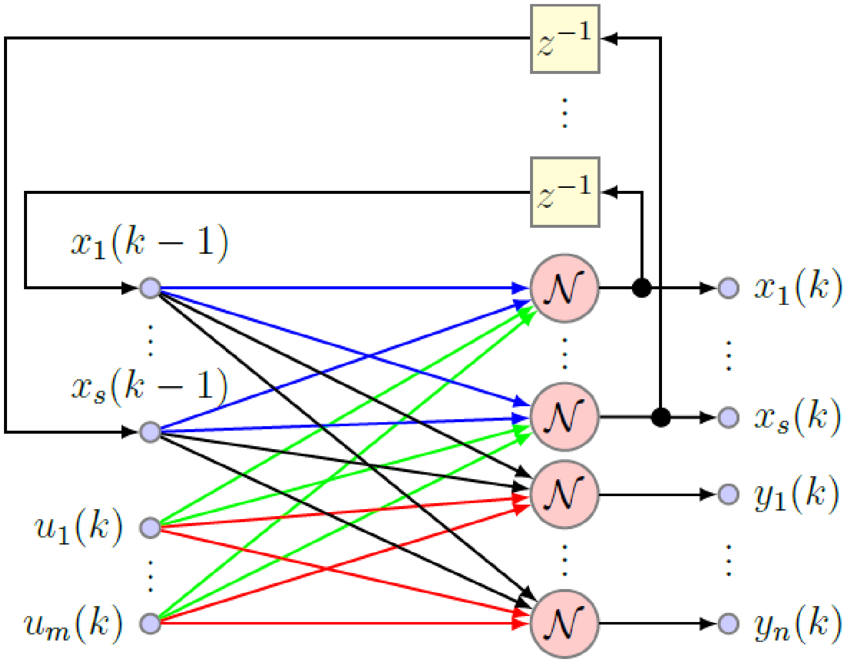
Flight data can be used to improve the physics-based model in a grey-box modeling scheme.



We would like to obtain a model that can perform multi-step predictions of the quadrotor behaviour by using motor inputs only

Data driven modeling – regression with RNN

- Recurrent Neural Networks (RNNs) are not only universal approximators but also have internal dynamics.
- considered strong candidate for accurate representation of dynamical systems.



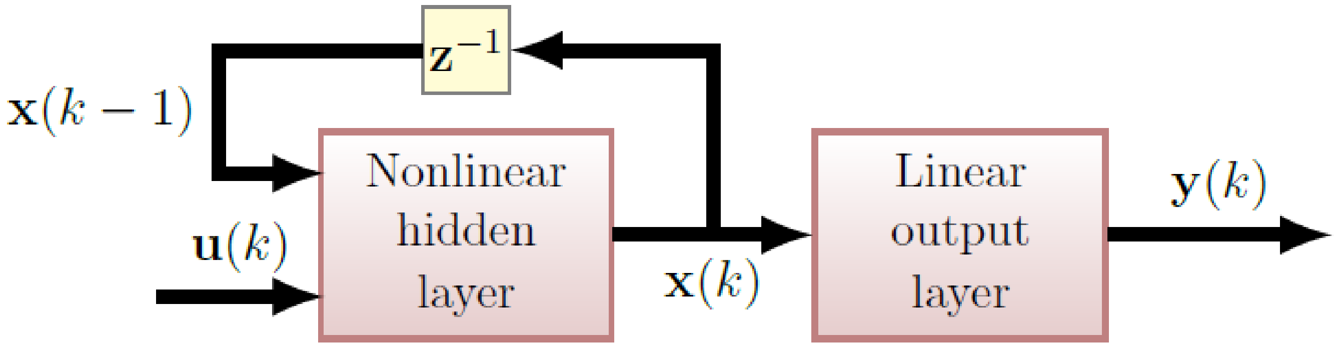
m inputs, s hidden and n output neurons

Data driven modeling – regression with RNN

Looks like a state-space representation

$$\mathbf{x}(k) = \mathbf{f}(\mathbf{A}\mathbf{x}(k-1) + \mathbf{B}\mathbf{u}(k) + \mathbf{b}_x)$$

$$\mathbf{y}(k) = \mathbf{g}(\mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) + \mathbf{b}_o).$$



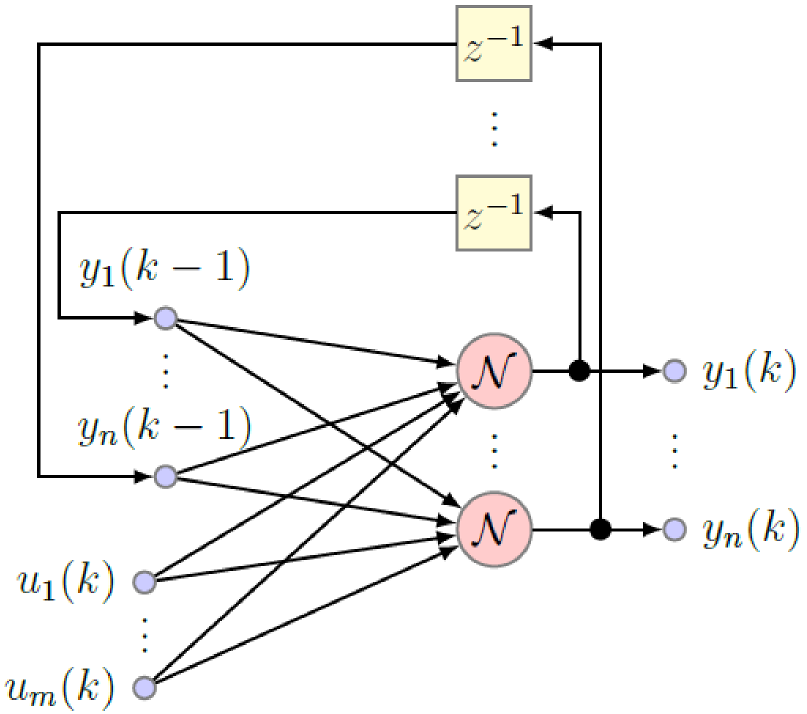
Data driven modeling – regression with RNN

Learning Algorithms for RNNs

1. Real Time Recurrent Learning :
    - ✓ network weights are continually updated as the network receives input elements
    - ✓ suitable when it is required to train the network while continually running it
2. Back Propagation Through Time :
    - ✓ gradient is calculated for a (finite) time horizon and any gradient based method can be applied to update the network weights.

Data driven modeling – regression with RNN

Learning Algorithms for RNNs : RTRL



$z_l(k)$ $y_i(k)$

$$v_i(k) = \sum_{l \in \mathcal{O} \cup \mathcal{I}} w_{il} z_l(k)$$

$$y_i(k) = f_i(v_i(k))$$

$$E(k) = \frac{1}{2} \sum_{i \in \mathcal{O}} e_i^2(k)$$

$$e_i(k) = y_i^d(k) - y_i(k)$$

Data driven modeling – regression with RNN

Learning Algorithms for RNNs : RTRL

cost function can be either the instantaneous error or a total error over a given period such as

$$E(k) = \frac{1}{2} \sum_{i \in \mathcal{O}} e_i^2(k) \qquad\qquad L = E(k)\Big|_{k=t_0}^{T+t_0} = \sum_{k=t_0}^{T+t_0} E(k).$$

$$\Delta w_{ij}(k) = -\eta \nabla_{\mathbf{W}} E(k) = -\eta \frac{\partial E(k)}{\partial w_{ij}} = -\eta \sum_{l \in \mathcal{O}} e_l(k) \frac{\partial y_l(k)}{\partial w_{ij}},$$

Multi-Step Prediction for Dynamic Systems

Input sequence of length T starting at a time instance k_0 + 1, U(k_0 + 1; T)

$$\mathbf{U}(k_0+1,T) = \begin{bmatrix}\mathbf{u}(k_0+1) & \mathbf{u}(k_0+2) & \ldots & \mathbf{u}(k_0+T)\end{bmatrix}$$

system response to this input is an output sequence denoted by Y(k_0 + 1; T)

$$\mathbf{Y}(k_0+1,T) = \begin{bmatrix}\mathbf{y}(k_0+1) & \mathbf{y}(k_0+2) & \ldots & \mathbf{y}(k_0+T)\end{bmatrix}$$

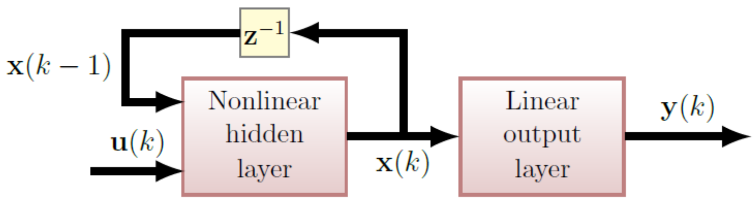
Our Problem is :

Given an input sequence U(k_0 + 1; T), the multi-step prediction problem seeks an accurate estimate of the system output, $\tilde{Y}$(k_0 + 1; T) , over the same time horizon, T

$$\tilde{\mathbf{Y}}(k_0+1,T) = \begin{bmatrix}\tilde{\mathbf{y}}(k_0+1) & \tilde{\mathbf{y}}(k_0+2) & \ldots & \tilde{\mathbf{y}}(k_0+T)\end{bmatrix}$$

Multi-Step Prediction for Dynamic Systems

$k=k_0+1$



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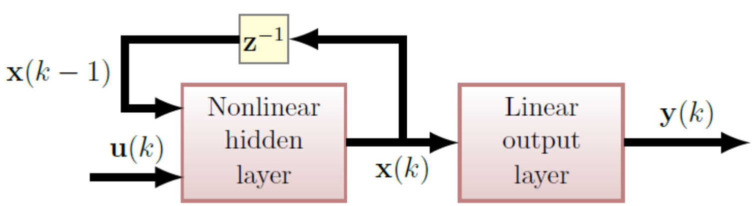
$k=k_0+T$



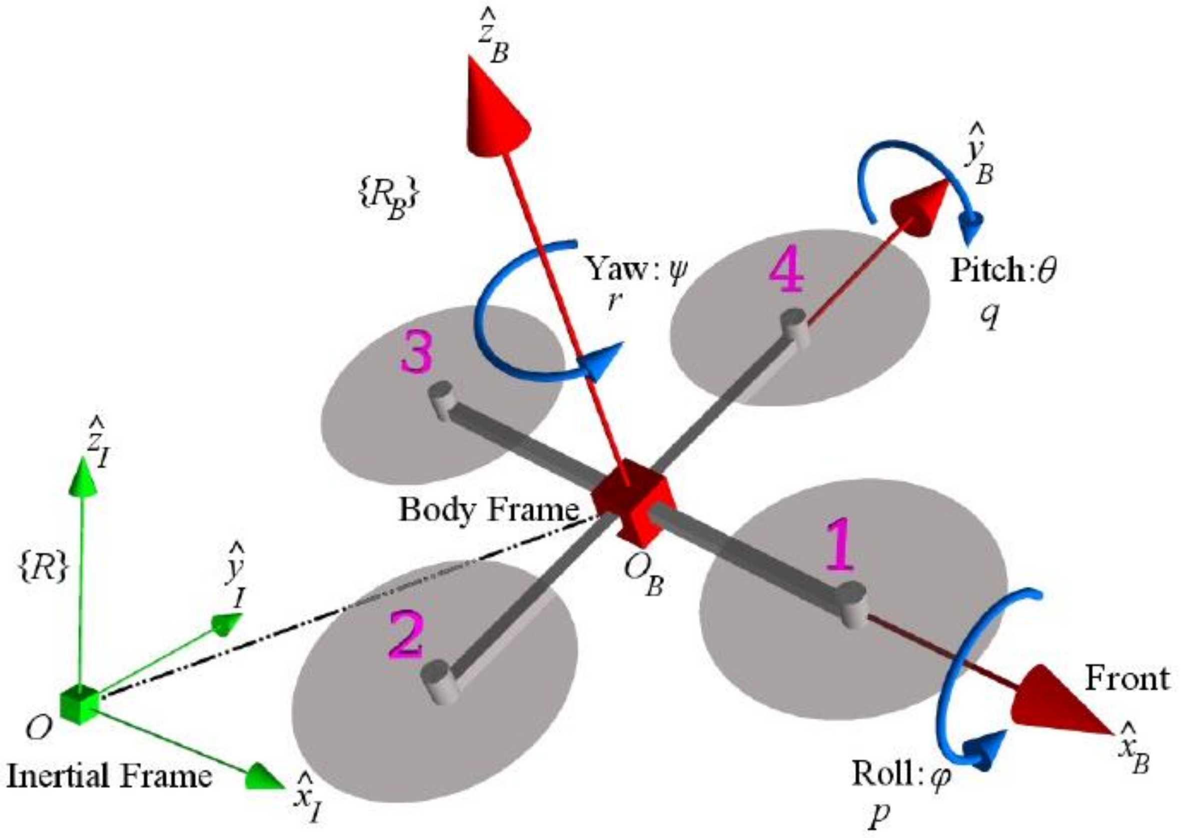
Cost function to be minimized :

$$L = \frac{1}{T} \sum_{k=k_0+1}^{k_0+T} \mathbf{e}(k)^\top \mathbf{e}(k),$$

$$\mathbf{e}(k) = \mathbf{y}(k) - \tilde{\mathbf{y}}(k).$$

Physics based modeling – Quadrotor Model

Quadrotor frames and variables



Physics based modeling – Quadrotor Model

- Physics based model of a quadrotor:

Euler angles

Body angular rates

$$\boldsymbol{\eta} = [\phi \quad \theta \quad \psi]^T, \boldsymbol{\omega}_B = [p \quad q \quad r]^T,$$

$$\boldsymbol{\xi}_I = [x_I \quad y_I \quad z_I]^T, \mathbf{v}_I = [x_I \quad y_I \quad z_I]^T$$

Position in inertial frame

Velocity in inertial frame

Rotation matrices

Inertia

$$\begin{bmatrix} \dot{\boldsymbol{\eta}} \\ \dot{\boldsymbol{\omega}}_B \\ \dot{\xi}_I \\ \dot{\mathbf{v}}_I \end{bmatrix} = \begin{bmatrix} \mathbf{R}_E \boldsymbol{\omega}_B \\ \mathbf{J}^{-1}(\boldsymbol{\tau}_\Theta - \boldsymbol{\omega}_B \times \mathbf{J}\boldsymbol{\omega}_B) \\ \mathbf{v}_I \\ \frac{1}{m}(\mathbf{R}_{B \to I} T_{tot} - k_t \, \mathbf{v}_I) - g \end{bmatrix}$$

Force to thrust mapping:

$$\begin{bmatrix} T_{tot} \\ \boldsymbol{\tau}_\Theta \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ l & 0 & -l & 0 \\ 0 & -l & 0 & l \\ -d & d & -d & d \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

i^{th} motor force: $f_i = k_i {\omega_i}^2$

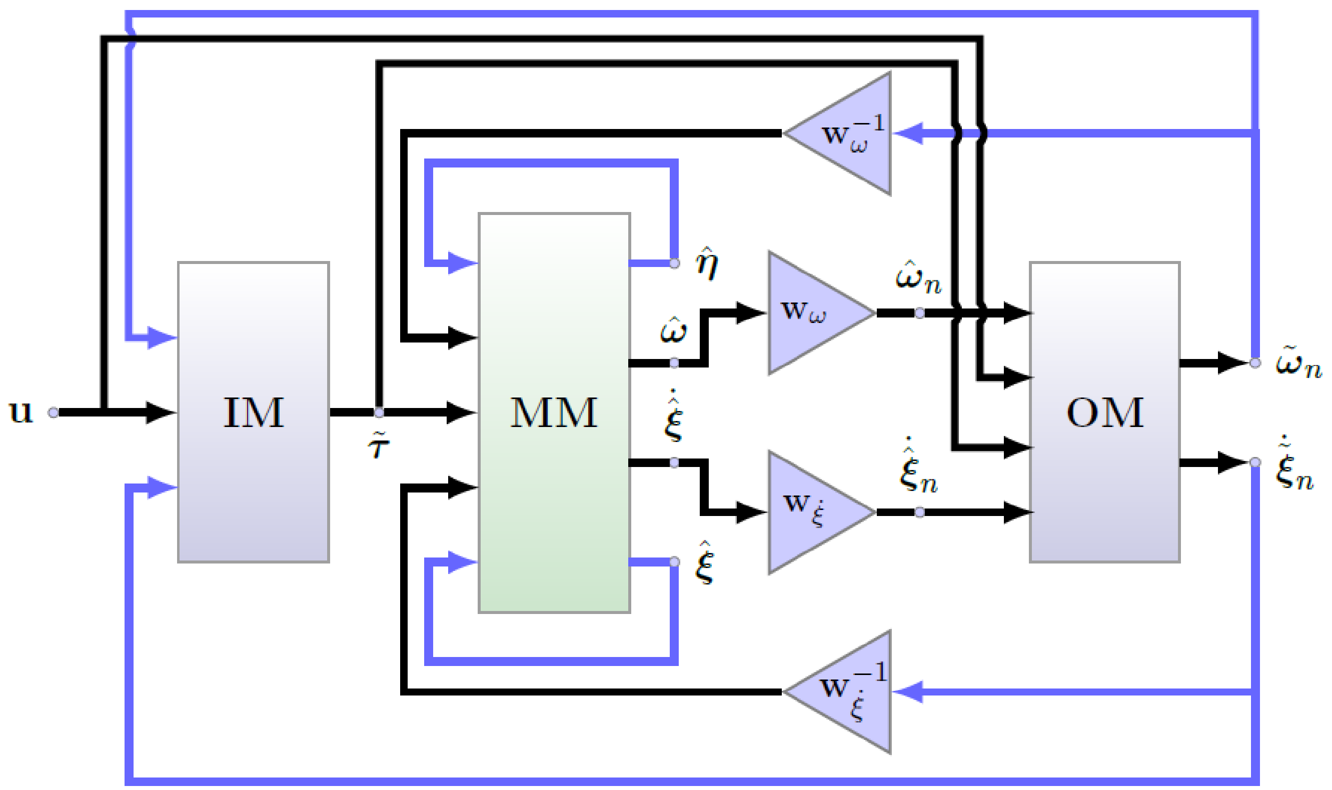
Hybrid Models for Multi-Step Prediction

- some characteristics of the system might be too difficult or expensive to accurately model, such as the vortex ring effect on a quadrotor
- A grey-box modeling approach can speed up the modeling process and increase the prediction accuracy of the model.

Hybrid Models for Multi-Step Prediction

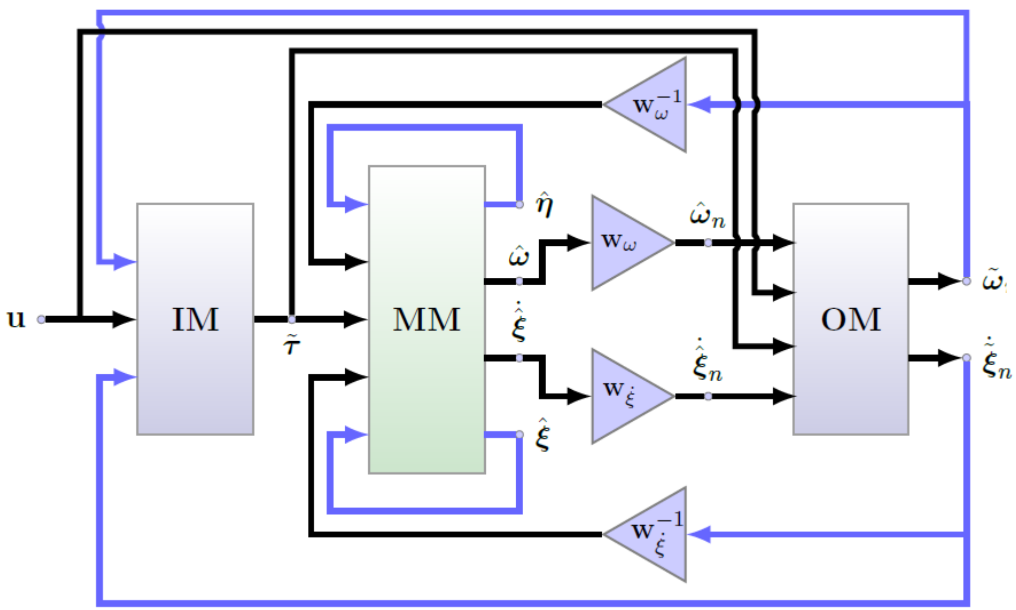
hybrid model consists of two black-box modules and a white-box module

- black-box modules : Input Model (IM) and Output Model (OM)
- white-box module : Motion Model (MM).



Hybrid Models for Multi-Step Prediction

- IM module generates the torques and thrust
- MM module updates the states of the quadrotor for one step using Equations
- OM module compensates for the prediction error introduced by the MM module because of the unmodeled dynamics and noise



Rotation matrices

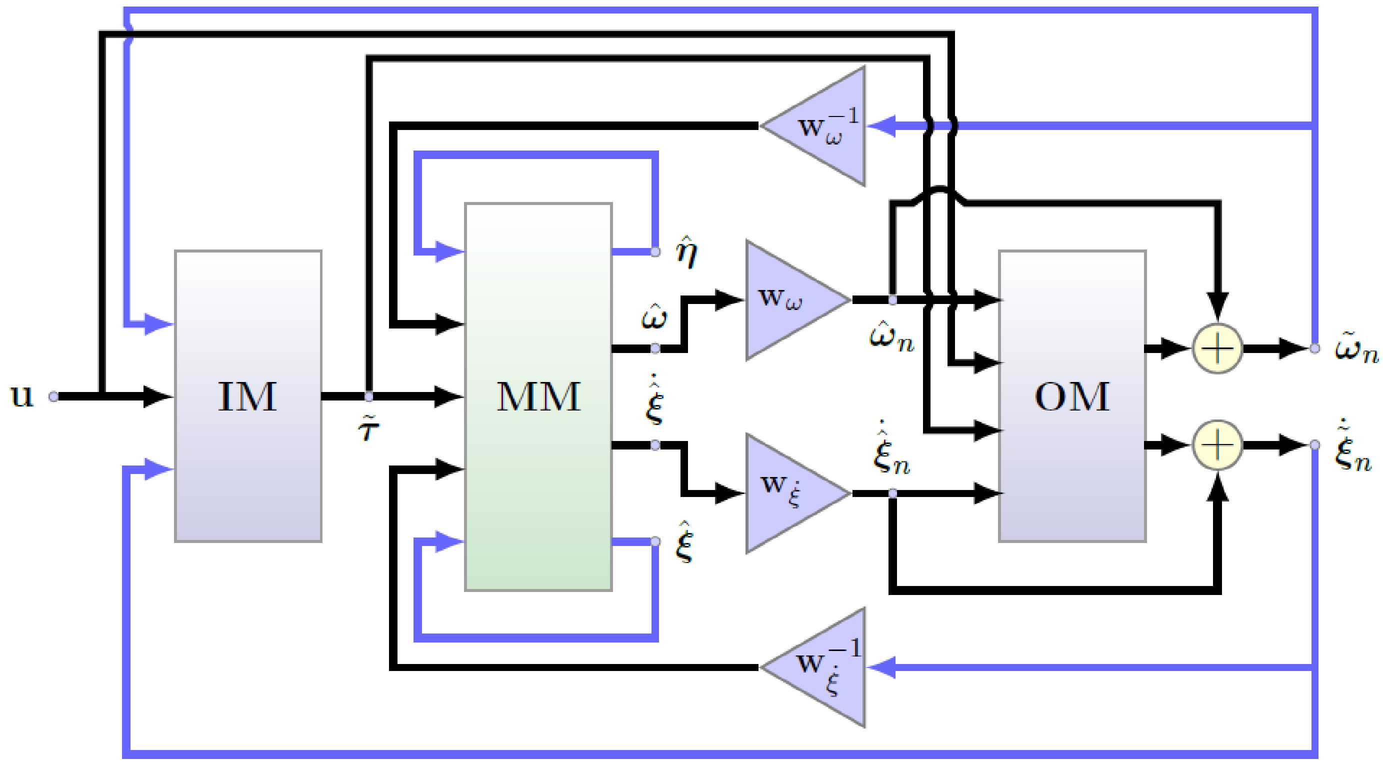
Inertia

$$\begin{bmatrix} \dot{\boldsymbol{\eta}} \\ \dot{\boldsymbol{\omega}}_B \\ \dot{\boldsymbol{\xi}}_I \\ \dot{\mathbf{v}}_I \end{bmatrix} = \begin{bmatrix} \mathbf{R}_E \boldsymbol{\omega}_B \\ \mathbf{J}^{-1}(\boldsymbol{\tau}_\Theta - \boldsymbol{\omega}_B \times \mathbf{J}\boldsymbol{\omega}_B) \\ \mathbf{v}_I \\ \frac{1}{m}(\mathbf{R}_{B\to I} T_{tot} - k_t \, \mathbf{v}_I) - g \end{bmatrix}$$

Hybrid Models for Multi-Step Prediction

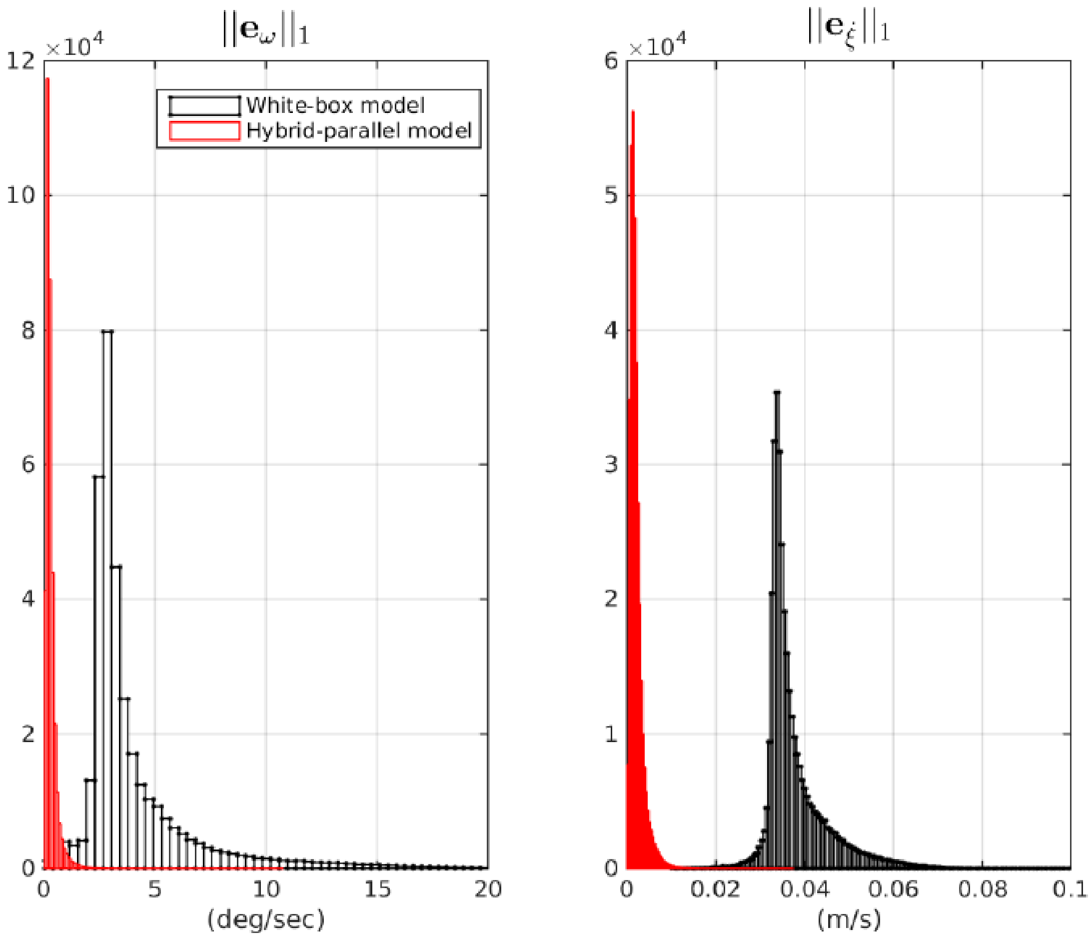
Parallel Configuration

OM module only account for the error from unmodeled dynamics and noise of MM module



Results

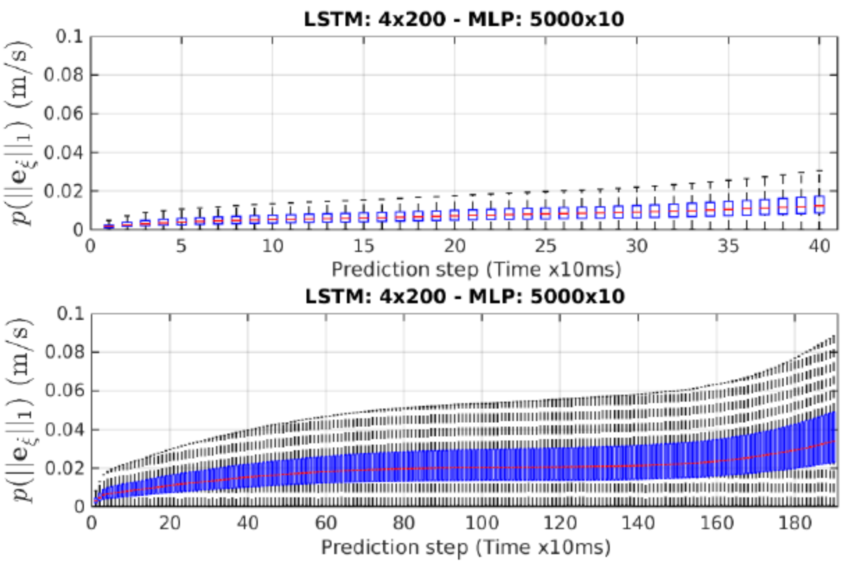
White-box model vs. hybrid-parallel model in a single-step prediction scenario



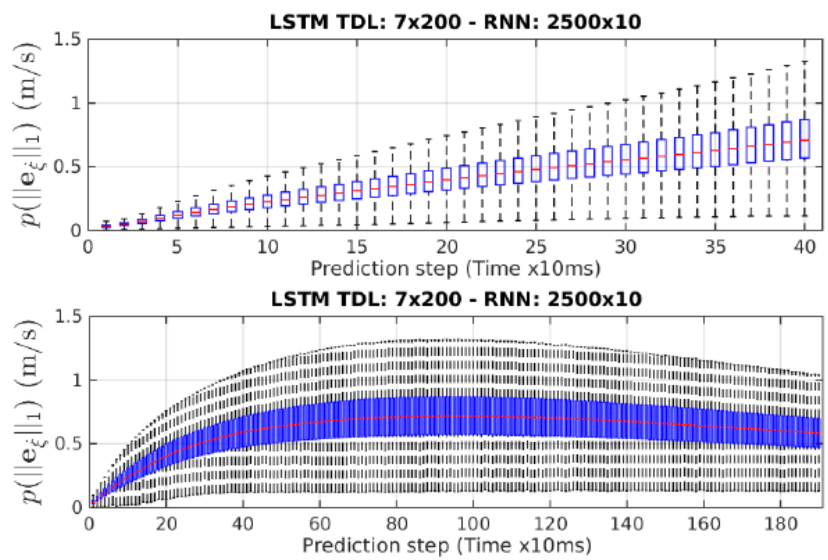
Results

Multi-step case :

An improvement more than an order of magnitude is observed by using the hybrid-parallel model



Hybrid



Black box